Received: *June 9, 1998,* Accepted: *July 29, 1998*

Probability amplitudes for charge-monopole scattering

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Abstract: In this letter we quantize a previously proposed non-local lagrangian for the classical dual electrodynamics [4], showing how it can be used to construct probability amplitudes. Our results are shown to agree with those obtained in the context of Schwinger and Zwanziger formulations, but without necessity of introducing strings.

Keywords: Duality in Gauge Field Theories, Solitons Monopoles and Instantons.

In spite of the remarkable role that magnetic monopoles have played in particle physics along the years as elementary particles in abelian dual electrodynamics [1] or topological solutions of non-abelian unified theories [2], a quantal approach for the interaction between charges and poles has been always a challenging open problem, due to two different difficulties [3]: the non-perturbative character of the charge-pole interaction; and the absence of a complete lagrangian formulation, connected to the impossibility of introducing regular 4-potentials.

In a previous work [4, 5], a covariant and gauge-invariant, manifestly dual, non-local lagrangian formalism has been reported, leading to the complete set of dual electromagnetic equations, without necessity of any subsidiary condition or constraint on the particles motion. Now, dismissing the problem of the non-perturbative value of the magnetic charge, we quantize such an approach, constructing an invariant perturbative theory for the charge-monopole interaction.

The referred non-local lagrangian, obeying a saddle-point action principle, has the interaction sector

$$\mathcal{L}_{int} = -j_\mu \mathcal{A}^\mu + g_\mu \tilde{\mathcal{A}}^\mu \,, \tag{1}$$

where j^μ and g^μ are, respectively, the electric and magnetic 4-currents, and the non-local potentials $\mathcal{A}^\mu$ and $\tilde{\mathcal{A}}^\mu$ are defined by

$$\mathcal{A}^\mu = A^\mu + \frac{1}{2}\epsilon^{\mu\gamma\alpha\beta} \int_P^x \partial_\alpha \tilde{A}_\beta \, d\xi_\gamma \,, \tag{2}$$

$$\tilde{\mathcal{A}}^\mu = \tilde{A}^\mu - \frac{1}{2}\epsilon^{\mu\gamma\alpha\beta} \int_{\tilde{P}}^x \partial_\alpha A_\beta \, d\xi_\gamma \,. \tag{3}$$

Here, A^μ and $\tilde{A}^\mu$ are the local potentials of Cabibbo and Ferrari [6], defined, in terms of the field strength, by

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - \epsilon^{\mu\nu\alpha\beta} \, \partial_\alpha \tilde{A}_\beta \tag{4}$$

and obeying, in Lorenz's gauge, the wave equations [7]

$$\partial^\nu \partial_\nu A^\mu = -j^\mu \,, \tag{5}$$

$$\partial^\nu \partial_\nu \tilde{A}^\mu = -g^\mu \,. \tag{6}$$

The interaction operator corresponding to (1) is given by [4]

$$V = -\frac{\partial S^e_{int}}{\partial t} + \frac{\partial S^g_{int}}{\partial t} = \int d^3x \, (j_\mu \mathcal{A}^\mu + g_\mu \tilde{\mathcal{A}}^\mu) \,, \tag{7}$$

where $S^{e;g}_{int}$ stand for the charge and monopole interaction actions, respectively. So we have, for the scattering matrix,

$$S = T \exp\left\{ -i \int d^4x \, (j_\mu \mathcal{A}^\mu + g_\mu \tilde{\mathcal{A}}^\mu) \right\} \,. \tag{8}$$

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Expanding in powers of e and g, the electric and magnetic charges, the first non-diagonal contribution is the second order one

$$S^{(2)} = -\frac{1}{2}\int\int d^4x\ d^4x'\ \{T[j^\mu(x)j^\nu(x')]T[\mathcal{A}_\mu(x)\mathcal{A}_\nu(x')] + \\ + T[g^\mu(x)g^\nu(x')]T[\tilde{\mathcal{A}}_\mu(x)\tilde{\mathcal{A}}_\nu(x')] + 2T[j^\mu(x)g^\nu(x')]T[\mathcal{A}_\mu(x)\tilde{\mathcal{A}}_\nu(x')]\} \tag{9}$$

The first and second terms correspond, respectively, to charge-charge and pole-pole scatterings. In the first case, we can use gauge invariance to put $\tilde{A}^\mu = 0$[9] and to introduce the photon propagation function

$$D_{\mu\nu}(x-x') = i\langle 0|T\mathcal{A}_\mu(x)\mathcal{A}_\nu(x')|0>= i<0|TA_\mu(x)A_\nu(x')|0\rangle \tag{10}$$

and, in the second case, putting $A^\mu = 0$ we can use the propagation function

$$\tilde{D}_{\mu\nu}(x-x') = i\langle 0|T\tilde{\mathcal{A}}_\mu(x)\tilde{\mathcal{A}}_\nu(x')|0\rangle = i\langle 0|T\tilde{A}_\mu(x)\tilde{A}_\nu(x')|0\rangle \tag{11}$$

leading to a pole-pole interaction completely analogous to the charge-charge one.

The last term in (9) corresponds to the charge-monopole scattering and suggests the introduction of the mixed propagation function

$$C_{\mu\nu}(x-x') = 2i\langle 0|T\mathcal{A}_\mu(x)\tilde{\mathcal{A}}_\nu(x')|0\rangle\,. \tag{12}$$

This allows us to write the scattering amplitude in the form

$$M = eg(\bar{u}_g\gamma^\mu u_g)C_{\mu\nu}(\bar{u}_e\gamma^\nu u_e)\,, \tag{13}$$

where u_g and u_e are, respectively, the pole and charge amplitudes.[1]

It is easy to see that $<0|TA_\mu(x)\tilde{A}_\nu(x')|0>=0$, due to the fact that A^μ and $\tilde{A}^\mu$ describe photons with opposite parities.[2] Thus, using (2) and (3) we obtain, from (12),

$$C_{\mu\nu}(x-x') = i\epsilon_{\mu\gamma\alpha\beta} <0|T\tilde{A}_\nu(x')\int_P^x \partial^\alpha\tilde{A}^\beta\ d\xi^\gamma|0\rangle - \\ -i\epsilon_{\nu\gamma\alpha\beta}\langle 0|TA_\mu(x)\int_{\tilde{P}}^{x'} \partial^\alpha A^\beta\ d\xi^\gamma|0\rangle\,. \tag{14}$$

Remembering that, in the classical limit, P ($\tilde{P}$) coincides with the charge (pole) world-line between $\xi = -\infty$ and $\xi = x$ (x'), it is a straightforward calculation to verify that, in (14), the chronological ordering operator T commutes with the integral and derivative operators. Then, we have

$$\begin{aligned} C_{\mu\nu}(x-x') &= i\epsilon_{\mu\gamma\alpha\beta}\int_P^x d\xi^\gamma\ \partial_\xi^\alpha\langle 0|T\tilde{A}_\nu(x')\tilde{A}^\beta(\xi)|0\rangle - \\ &\quad -i\epsilon_{\nu\gamma\alpha\beta}\int_{\tilde{P}}^{x'} d\xi^\gamma\ \partial_\xi^\alpha\langle 0|TA_\mu(x)A^\beta(\xi)|0\rangle \\ &= \epsilon_{\mu\gamma\alpha\beta}\int_P^x d\xi^\gamma\ \partial_\xi^\alpha\tilde{D}_\nu^\beta(x'-\xi) - \epsilon_{\nu\gamma\alpha\beta}\int_{\tilde{P}}^{x'} d\xi^\gamma\ \partial_\xi^\alpha D_\mu^\beta(x-\xi) \\ &= \epsilon_{\mu\gamma\alpha\beta}\int_P^{x-x'} d\xi^\gamma\ \partial_\xi^\alpha\tilde{D}_\nu^\beta(\xi) - \epsilon_{\nu\gamma\alpha\beta}\int_{\tilde{P}}^{x'-x} d\xi^\gamma\ \partial_\xi^\alpha D_\mu^\beta(\xi) \end{aligned} \tag{15}$$

[1]To fix ideas, we are considering charge and pole as 1/2-spin particles.

[2]Actually, this reasoning is not necessary: by fixing the gauges $A^\mu = 0$ or $\tilde{A}^\mu = 0$, this expectation value vanishes trivially.

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Thus, we can write the mixed propagator in terms of the propagators (10), (11) as

$$C_{\mu\nu}(x) = \epsilon_{\mu\gamma\alpha\beta}\int_P^x d\xi^\gamma\ \partial_\xi^\alpha \tilde{D}_\nu^\beta(\xi) - \epsilon_{\nu\gamma\alpha\beta}\int_{\tilde{P}}^{-x} d\xi^\gamma\ \partial_\xi^\alpha D_\mu^\beta(\xi)\,. \tag{16}$$

Once more, we can use the gauges $A_\mu = 0$ or $\tilde{A}_\mu = 0$ in order to obtain, respectively,

$$C_{\mu\nu}(x) = \epsilon_{\mu\gamma\alpha\beta}\int_P^x d\xi^\gamma\ \partial_\xi^\alpha \tilde{D}_\nu^\beta(\xi) \tag{17}$$

and

$$C_{\mu\nu}(x) = -\epsilon_{\nu\gamma\alpha\beta}\int_{\tilde{P}}^{-x} d\xi^\gamma\ \partial_\xi^\alpha D_\mu^\beta(\xi)\,. \tag{18}$$

We see that the mixed propagator (and then the amplitude (13)) is a non-local quantity, depending on the integration paths P and $\tilde{P}$. Nevertheless, this non-locality, like in the classical case, will be shown non-observable when we calculate observable quantities like $|M|^2$ and the scattering cross section.

Indeed, these observables can be obtained if we calculate, from (17), the local quantity[3]

$$\partial_\lambda C_{\mu\nu}(x) = \epsilon_{\mu\lambda\alpha\beta}\partial^\alpha \tilde{D}_\nu^\beta(x) \tag{19}$$

Then we see that the mixed propagator obeys the equation

$$\partial^\lambda\partial_\lambda C_{\mu\nu} = \epsilon_{\mu\nu\lambda\alpha}[\partial^\lambda, \partial^\alpha] D_F(x)\,, \tag{20}$$

where we have used $\tilde{D}^{\beta\nu}(x) = g^{\beta\nu} D_F(x)$.[4]

In the momentum representation, (19) has the form

$$k_\lambda C_{\mu\nu}(k) = \epsilon_{\mu\lambda\alpha\beta} k^\alpha \tilde{D}_\nu^\beta(k)\,. \tag{21}$$

Using for the photon propagator

$$\tilde{D}^{\mu\nu}(k) = \frac{4\pi}{k^2} g^{\mu\nu}\,, \tag{22}$$

we have

$$k_\lambda C_{\mu\nu}(k) = \frac{4\pi}{k^2} k^\alpha \epsilon_{\mu\nu\lambda\alpha}\,. \tag{23}$$

It is easy to show that using (18) leads to the same result, showing that the exchange of the two kinds of photons present in the theory (the fields A^μ and $\tilde{A}^\mu$) gives identical contributions to observable quantities, the two kinds of photons being indistinguishable from the observational point of view. This is related to the fact that, in the classical version of the theory, introducing the additional 4-potential $\tilde{A}^\mu$ does not change the

[3] See footnote 2 in [4].

[4] It is important to remark that the commutator $[\partial^\lambda, \partial^\alpha] D_F(x)$ does not vanish in all space-time, due to the discontinuity of the Feynman propagator $D_F(x)$ at $t = 0$.

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number of independent physical degrees of freedom, due to the presence of extra gauge invariance [9].

From (23) and (13), we can derive

$$k_\lambda M = eg(\bar{u}_g\gamma^\mu u_g)k_\lambda C_{\mu\nu}(\bar{u}_e\gamma^\nu u_e) = \frac{4\pi eg}{k^2}k^\alpha\epsilon_{\mu\nu\lambda\alpha}(\bar{u}_g\gamma^\mu u_g)(\bar{u}_e\gamma^\nu u_e)\,. \tag{24}$$

Therefore,

$$|k_\lambda M|^2 = k^2|M|^2 = \frac{32e^2g^2\pi^2}{k^4}U_{\mu\nu}U^\dagger_{\alpha\beta}k_\sigma k_\delta\epsilon^{\mu\nu\lambda\sigma}\epsilon_\lambda{}^{\alpha\beta\delta} \tag{25}$$

or

$$|M|^2 = \frac{32e^2g^2\pi^2}{k^6}U_{\mu\nu}U^\dagger_{\alpha\beta}k_\sigma k_\delta\epsilon^{\mu\nu\lambda\sigma}\epsilon_\lambda{}^{\alpha\beta\delta}\,, \tag{26}$$

where we have introduced

$$U^{\alpha\beta} \equiv (\bar{u}_g\gamma^\alpha u_g)(\bar{u}_e\gamma^\beta u_e)\,. \tag{27}$$

From (26), we see that only the antisymmetric part of $U_{\mu\nu}$ and $U^\dagger_{\alpha\beta}$ will contribute to $|M|^2$. A direct calculation leads us to

$$\frac{1}{64\pi^2}|M|^2 = \frac{e^2g^2}{k^4}U^{[\mu\nu]}U^\dagger_{[\mu\nu]}\,, \tag{28}$$

where $U^{[\mu\nu]}$ indicates the antisymmetric part of $U^{\mu\nu}$, and where we have used the transversality conditions $k^\mu j_\mu = k^\mu g_\mu = 0$, expressing the conservation of the electric and magnetic currents in the momentum representation.

At this point it is interesting to compare our results with those obtained by Dirac-type string formulations. On the basis of Schwinger's formalism [10], Rabl [11] derived the gauge-dependent propagator

$$C'_{\mu\nu}(k) = \frac{4\pi}{k^2}\frac{\epsilon_{\mu\nu\lambda\alpha}n^\lambda k^\alpha}{(n\cdot k)} \tag{29}$$

where n^λ is a fixed but arbitrary 4-vector pointing in the string direction. This propagator was also obtained by Zwanziger [12] in the context of another string-based formulation.[5]

Equation (29) can be rewritten as

$$n^\lambda(k_\lambda C'_{\mu\nu}) = n^\lambda\left(\frac{4\pi}{k^2}k^\alpha\epsilon_{\mu\nu\lambda\alpha}\right) \tag{30}$$

and, as n^λ is an arbitrary 4-vector, we have

$$k_\lambda C'_{\mu\nu} = \frac{4\pi}{k^2}k^\alpha\epsilon_{\mu\nu\lambda\alpha}\,, \tag{31}$$

which must be compared to (23).

[5]For a good review on the Schwinger and Zwanziger approaches, see [3].

On the other hand, multiplying (23) by an arbitrary 4-vector n^λ, we can put it in the form

$$C_{\mu\nu}(k) = \frac{4\pi}{k^2} \frac{\epsilon_{\mu\nu\lambda\alpha} n^\lambda k^\alpha}{(n \cdot k)} \,. \tag{32}$$

From (29) and (32) we see that $C_{\mu\nu} = C'_{\mu\nu}$, that is, our mixed propagator is equal to the propagator derived in the Schwinger and Zwanziger formulations.

While equation (16) exhibits the non-local character of the mixed propagator, equation (32) (being dependent on the arbitrary 4-vector n^λ) shows its gauge-dependence [3]. Actually, the relation between non-locality and gauge-dependence is already present in the classical version of the theory: a change of the paths of integration P and $\tilde{P}$ leads only to a gauge transformation of the non-local potentials (2) and (3) [13]. By the way, let us emphasize that it is this property that guarantees the full covariance of the formalism and the strict locality of observables and equations of motion.

Let us calculate the classical limit of the probability (28), obtaining the differential cross section for the elastic, non-relativistic, scattering of an electron by a massive monopole at rest. In the classical limit, the electric and magnetic 4-currents are given by

$$j^\mu = e\bar{u}_e \gamma^\mu u_e = e\gamma_e(1, \vec{v}_e) \,, \tag{33}$$

$$g^\mu = g\bar{u}_g \gamma^\mu u_g = g\gamma_g(1, \vec{v}_g) \,, \tag{34}$$

where $\gamma_{e,g} = (1 - v_{e,g}^2)^{-1/2}$ stand for the Lorentz factors for the charge and pole, respectively.

By using these currents in (28) we obtain, in the monopole rest frame,

$$\frac{1}{16\pi^2} |M|^2 = \frac{e^2 g^2 v_e^2 \gamma_e^2}{k^4} \,, \tag{35}$$

which leads, in the non-relativistic limit, for a small angle of scattering, to the differential cross section

$$d\sigma = \frac{e^2 g^2 v_e^2}{|\vec{p}|^4 \theta^4} d\Omega \,, \tag{36}$$

where $\vec{p}$ is the momentum of, say, the incident electron.

This result was originally obtained by Goldhaber in the context of a non-relativistic approach [11, 14]. It can be interpreted as the cross section for the Coulomb scattering of a charge e by a charge gv_e, in accordance with the fact that, in the classical theory, a static charge-pole pair does not interact. By the way, let us note that the dependence on the relative velocity explains the obtainment of (36) in the context of a perturbative expansion, even the magnetic charge being non-perturbative: in this non-relativistic limit, the effective charge $gv_e << 1$.

Finally, let us briefly comment the question of *dyons*. The formalism proposed in [4] is invariant only under the discrete dual transformation corresponding to a dual angle of $\pi/2$, i.e., the transformation that interchanges the electric and magnetic charges; it is not invariant under a general dual transformation with arbitrary dual angle, what means in particular that our lagrangian is not appropriate to describe elementary dyons.

This is intimately connected to the saddle-point character of the action on which the formalism is based: an elementary particle cannot simultaneously minimize (as an electric charge) and maximize (as a monopole) the action. Of course nothing forbids one to describe dyons as composite systems but, if we want to describe them as elementary ones, some generalization of the theory is needed.

Acknowledgments

I would like to thank Drs. E. Ferrari, J. Frenkel, J.A. Helayel-Neto, E. Recami and F. Rohrlich for usefull discussions and suggestions. My thanks also to Dr. C.O. Escobar for the hospitality in *Universidade de São Paulo* during the elaboration of this paper and to Prof. N. Andion for the reading of the manuscript.

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[7] In reference [4], Maxwell's equations are obtained by varying the action with respect to the non-local potentials (2) and (3). In a recent paper [8], it has been suggested that varying the action with respect to the local potentials leads to extra terms like

$$h_\mu(x) = \frac{1}{2}\epsilon_{\mu\gamma\alpha\beta}\int d^4y\; g^\gamma(y)\;\left[\int_{\tilde{P}}^{y}\partial_x^\alpha\delta^4(x-\xi)d\xi^\beta\right].$$

Nevertheless, it is possible to verify that such extra terms are identically null. Indeed, integrating by parts, we can rewrite the above quantity as[6]

$$\begin{aligned} h_\mu(x) &= \frac{1}{2}\epsilon_{\mu\gamma\alpha\beta}\int dS^{\beta\lambda\sigma}\;\left[\int_{\tilde{P}_S}^{y}\partial_x^\alpha\delta^4(x-\xi)d\xi_\lambda\right]\;\left[\int_{\tilde{P}_S}^{y} g^\gamma(\chi)d\chi_\sigma\right] \\ &-\frac{1}{2}\epsilon_{\mu\gamma\alpha\beta}\int d^4y\;\left[\partial_x^\alpha\delta^4(x-y)\right]\;\left[\int_{\tilde{P}}^{y} g^\gamma(\xi)d\xi^\beta\right] \end{aligned}$$

[6] See footnote 2 in [4].

being $\tilde{P}_S$ on the hypersurface at infinity. Clearly, the hypersurface term is null for any finite x and so it does not contribute to the action variation. Using $\int f(x)\delta'(x)dx = -\int \delta(x)f'(x)dx$, the remaining term can be written as

$$\begin{aligned} h_\mu(x) &= -\frac{1}{2}\epsilon_{\mu\gamma\alpha\beta}\int d^4y\ \delta^4(x-y)\ \left[\partial_y^\alpha \int_{\tilde{P}}^y g^\gamma(\xi)\ d\xi^\beta\right] \\ &= -\frac{1}{2}\epsilon_{\mu\gamma\alpha\beta}\ \delta^{\alpha\beta}\int d^4y\ g^\gamma(y)\ \delta^4(x-y) \\ &= -\frac{1}{2}\epsilon_{\mu\gamma\alpha\beta}\ \delta^{\alpha\beta}\ g^\gamma(x) \equiv 0 \end{aligned}$$

due to the antisymmetry of $\epsilon_{\mu\gamma\alpha\beta}$. I would like to thank Dr. N. Berkovits for interesting discussions about this point.

[8] N. Berkovits, *Phys. Lett.* **B 395** (1997) 28 [hep-th/9610134].

[9] The field strength (4) is invariant under the generalized gauge transformations

$$\begin{aligned} A_\mu &\to A_\mu + A'_\mu \\ \tilde{A}_\mu &\to \tilde{A}_\mu + \tilde{A}'_\mu \end{aligned}$$

provided A'_μ and $\tilde{A}'_\mu$ satisfy the zero field condition

$$\partial^\mu A'^\nu - \partial^\nu A'^\mu - \epsilon^{\mu\nu\alpha\beta}\ \partial_\alpha \tilde{A}'_\beta = 0\,.$$

These gauge transformations generalize the usual ones

$$\begin{aligned} A_\mu &\to A_\mu + \partial_\mu \Lambda \\ \tilde{A}_\mu &\to \tilde{A}_\mu + \partial_\mu \Gamma \end{aligned}$$

for which the zero field condition is identically satisfied. As discussed in references [3] and [6], due to this extended gauge invariance, the introduction of the new potential $\tilde{A}_\mu$ does not increase the number of independent physical degrees of freedom of the electromagnetic field.

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[13] In fact, let us consider a magnetic monopole at rest in the origin of a coordinate system (the electric charge distribution can be arbitrary). The field equation (6) will have the Coulomb solution

$$\begin{aligned} \vec{\tilde{A}} &= 0 \\ \tilde{A}^0 &= \frac{g}{4\pi r}\,. \end{aligned}$$

Taking this solution in (2) we obtain

$$\begin{aligned} \mathcal{A}^0 &= A^0 \\ \vec{\mathcal{A}}(\vec{r}) &= \vec{A}(\vec{r}) - \frac{g}{8\pi}\int_P^{\vec{r}} \vec{\nabla}\left(\frac{1}{r'}\right) \times d\vec{r}'\,. \end{aligned}$$

If we take a different integration path P', the new vector potential $\vec{\mathcal{A}}'$ will differ from $\vec{\mathcal{A}}$ by

$$\vec{\mathcal{A}}'(\vec{r}) - \vec{\mathcal{A}}(\vec{r}) = \frac{g}{8\pi} \oint_{P-P'} \vec{\nabla}\left(\frac{1}{r'}\right) \times d\vec{r}' = \frac{g}{8\pi} \vec{\nabla}\Omega_0(\vec{r}) \,,$$

where $\Omega_0(\vec{r})$ is the solid angle formed by the contour $P - P'$ in the origin. So, the two considered non-local potentials differ by the gradient of a scalar function. Using now the covariance and linearity of (2), (5) and (6) we see that the 4-potentials $\mathcal{A}'_\mu$ and $\mathcal{A}_\mu$ differ by a gauge transformation in any frame of reference and whatever the distribution of charges and poles, as we wished to prove. This line of reasoning can also be applied to the dual non-local potential $\tilde{\mathcal{A}}_\mu$.

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